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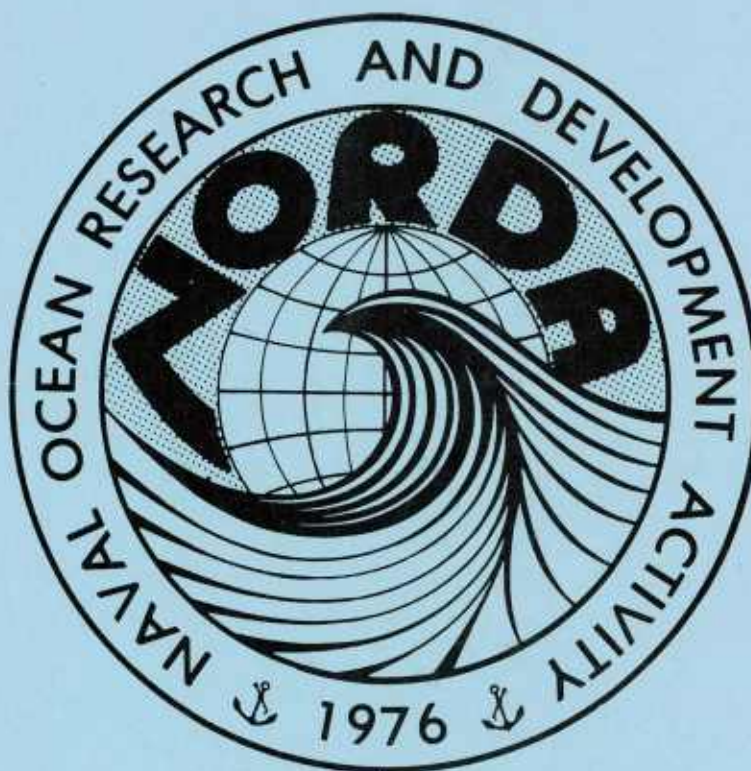
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Naval Ocean Research and
Development Activity
NSTL, Mississippi 39529



Analytical Modeling of Random Thermal Fluctuations in the NORDA High-Frequency Sound Scattering Facility



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Allan D. Pierce
Atlanta, Georgia
July 1984

ABSTRACT

An analytical study is presented of the laminar and turbulent flow fields that result from natural convection created by cooled or heated arrays of tubular elements immersed in water. The study is concerned in particular with the methods used to create turbulence in the NORDA high-frequency sound scattering facility and many numerical results are given that pertain to that application. The report discusses the initially laminar thermal plume that rises from a line source of heat in water, the instability of this plume, the eventual turbulent form of the plume, plume-plume interactions, and the swaying of thermal plumes. An appendix gives a chronological bibliography of works on natural convection that may be pertinent to understanding of turbulent flows created in situations analogous to the NORDA high-frequency scattering facility.

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I. INTRODUCTION

The tasks required in the subject contract are:

- (1) Develop a predictive model for the principal features of the sound speed fluctuations and flow fields that result from cooled or heated arrays of tubular elements immersed in water.
- (2) Apply the model to the NORDA high frequency scattering facility.
- (3) Document (1) and (2) in sufficient detail to permit NORDA personnel to incorporate the results into their research on high frequency propagation.

The NORDA high frequency scattering facility has been described by Levenson and Posey [1] and by Posey and Levenson [2,3] in three papers presented at recent meetings of the Acoustical Society of America. A fourth paper by Posey, Levenson, Branch, and Carver [4] discussed the thermal microstructure measurement system (TMMS) that is used in this facility. The water tank in this facility (see Fig. 1) is 14 m long by 3.7 m wide by 3.7 m deep. In typical experiments a 1.5-MHz cw acoustic signal is propagated over a 10 m horizontal path within the tank. A principal objective has been to study the effects of random sound speed fields on high-frequency propagation. To attain the desired random sound fields the NORDA experimenters have either cooled the water near the tank's top or heated the water near the tank's bottom. In either case natural convection was being used to create an inhomogeneous temperature field within the water.

A basic theory that the water tank experiments should test is that devised by Wenzel [5]. The simplest version of this theory requires ideally that the thermal fluctuations along and near the propagation path have statistical properties of a

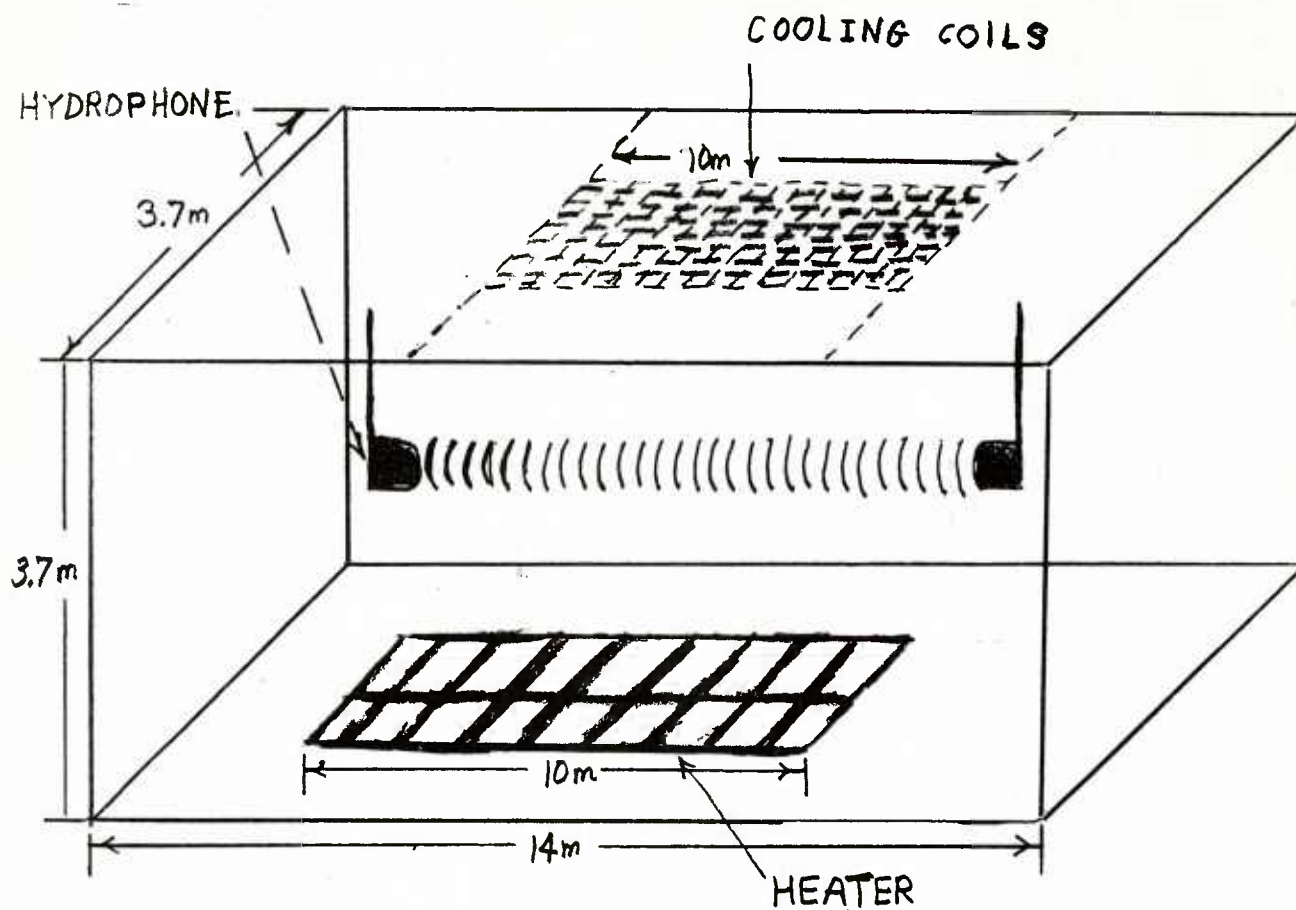


Figure 1. Sketch of a typical experimental configuration used for the study of the propagation of high frequency sound through turbulence in the NORDA water tank facility.

homogeneous stationary random process. Also, the test of the theory would be easier to make if there is no preferred direction (isotropic turbulence) in the statistical properties of this field. In such cases there are two important parameters that enter into Wenzel's theory. One of these is the ratio v^2 of the mean squared deviation of the sound speed c from its average value c_0 to the square of the average sound speed. The other parameter is the correlation length ℓ , defined such that the product ℓc_0^2 is the line integral over separation distance (from zero to infinity) of the spatial autocorrelation function of the sound speed deviation. Wenzel assumes that this correlation length ℓ is large compared with either $1/k_0$ or $(d/k_0)^{1/2}$, where k_0 (equal to $2\pi f/c_0$) is the acoustic wavenumber and d is the propagation distance.

A principal prediction of Wenzel's theory is the phenomenon of saturation, whereby the second moment of the incoherent part of the acoustic signal asymptotically varies with propagation distance in accord with energy conservation notions. The order of magnitude of the propagation distances that characterize the transition to the saturation region is of the order of $(v^2 k_0^2 \ell)^{-1}$. Consequently, the situation in the corresponding NORDA water tank experiments should ideally be such that the length d of the propagation path is comparable to or larger than $(v^2 k_0^2 \ell)^{-1}$. With the speed of sound in water taken as 1500 m/s, with the acoustic frequency taken as 1.5-MHz, and with the propagation distance d taken as 10 m, the acoustic wavenumber k_0 is 6300 m^{-1} , its reciprocal $1/k_0$ is 0.16 mm,

while $(d/k_o)^{1/2}$ is 0.04 m. Consequently, one desires that the correlation length λ be much larger than 0.04 m and that the product $v^2 \lambda$ be at least as large as $1/(k_o^2 d)$, the latter being 2.5×10^{-9} m.

The experiments reported by Posey and Levenson have confirmed Wenzel's theory in most particulars, but up to now the "strength" of the turbulence has not been sufficient for observation of the saturation phenomenon. Consequently, the present author has undertaken to broadly study the principles that govern the generation of this turbulence.

II. BOUSSINESQ EQUATIONS FOR NATURAL CONVECTION

Although the intended application is to acoustics, the subject matter of this study is natural convection in an enclosed water tank, either heated from below or cooled from above. There is an extensive literature on this subject, much of which pertains indirectly to the tasks of the research described in this report. Consequently, a bibliography is included here as Appendix A.

The usual equations taken as a starting point in natural convection studies are the Boussinesq equations, which constitute an approximation to the general fluid dynamic equations. A modern derivation with extensive references to alternative derivations is given by Gray and Giorgini [6].

The general form of the Boussinesq equations adopted in the present study is as follows:

$$\nabla \cdot \vec{v} = 0 \quad (1a)$$

$$\rho D\vec{v}/Dt = -\nabla p^* + \mu \nabla^2 \vec{v} + g\beta\rho\theta \vec{e}_z \quad (1b)$$

$$\rho c_p D\theta/Dt = \kappa \nabla^2 \theta \quad (1c)$$

where \vec{v} is the fluid velocity, ρ is the ambient density, μ is the viscosity, θ is the deviation of the temperature from its ambient value, g is the acceleration due to gravity, \vec{e}_z is the unit vector in the vertical direction, β is the coefficient of thermal expansion, c_p is the specific heat at constant pressure, and κ is the thermal conductivity. The reduced pressure p^* is the actual pressure p plus $\rho g \Delta z$, where height Δz is measured relative to any convenient reference height. The operator D/Dt is $\partial/\partial t + \vec{v} \cdot \nabla$. For simplicity, ρ , μ , g , β , c_p , and κ are regarded as constant.

III. DIMENSIONLESS GROUPS

There are two general fundamental models that can be taken as a starting point in the analysis of natural convection phenomena that occur in the NORDA water tank:

- (1) Horizontal cylinder model. A long horizontal cylinder immersed in water has a different temperature

than the surrounding water. In the first stage of the analysis the water environment is regarded as unbounded.

- (2) Line source model. The source (or sink) of heat is not explicitly described but is regarded as extending along a horizontal line (analogous to the cylinder axis in the horizontal cylinder model). The source is characterized by the amount of heat that is being added to the fluid per unit length of line. If the fluid is being cooled then the source is actually a heat sink and the amount of heat being added per unit length is a negative number. Here also, in the first stage of the analysis, the fluid is regarded as unbounded.

The horizontal cylinder model is more specific than the line source model and is pertinent to the present study because the devices that have previously been used to heat or cool the NORDA tank have consisted of either (i) electrically conducting copper rods bent into zig-zag configurations, (ii) a sequence of parallel cylindrical probes, each mounted perpendicular to a long bus that conducts electrical current to the probes, or (iii) cylindrical tubes carrying liquid nitrogen, the tubes being coiled into a horizontal configuration resembling a string of continuous zeroes. Near the surfaces of the rods or tubes at distances somewhat shorter than any length scale characterizing the zig-zags or coils, one would expect the horizontal cylinder model to be more appropriate.

At radial distances from a horizontal cylinder that are large compared with the cylinder's radius R one would expect to get results fully equivalent to those predicted by a line source model, provided one makes an appropriate identification for the

heat generated per unit length by the cylinder. The line source model, moreover, may be an appropriate idealization for a heating element configuration as a whole, provided one limits predictions to distances somewhat greater than the element width and provided the element is substantially longer than it is wide.

In the most recent experiments reported by Posey and Levenson [3], the heating was caused by the situation (ii) above, each probe being 14 inch (35.6 cm) long with a net power loss per probe of 1210 watt. There were two intermeshed staggered rows of probes, 36 probes on each row, the probes being 11 inch or 27.9 cm apart, 72 probes in all extending over a net distance of 10 m. Thus in the region local to any one particular probe one might use a line source model with energy input of 3400 watts/m. Alternatively at great distances above the heating element it might be appropriate to make estimates using a line source model with energy input of 8700 watts/m.

In either case, horizontal cylinder or line source, one can regard the convection phenomena governed by the Boussinesq equations (1) and the corresponding boundary conditions as being described in terms of dimensionless quantities.

Horizontal Cylinder Model

If the source of convection is a horizontal cylinder of radius R whose temperature is θ_c degrees greater (or lesser if θ_c is negative) than ambient, the Boussinesq equations are readily cast in a dimensionless form. Velocities are expressed in units of $\mu/\rho R$, distances in units of R , time in units of $\rho R^2/\mu$, θ in units of θ_c , and p^* in units of $\mu^2/\rho R^2$. In this formulation only two distinct similitude parameters appear; these can be

taken as the Grashof number Gr and the Prandtl number Pr , defined as

$$Gr = g\beta (2R)^3 |\theta_c| / (\mu/\rho)^2 \quad (2a)$$

$$Pr = \mu c_p / K \quad (2b)$$

A convenient derived dimensionless group is the Rayleigh number, defined as

$$Ra = Gr \cdot Pr \quad (2c)$$

It is undoubtedly appropriate to have some estimates of the orders of magnitudes of these parameters at the outset. Taking values for water appropriate to 20°C , one has

$$\begin{aligned} g &= 9.8 \text{ m/s}^2 \\ \beta &= 2.07 \times 10^{-6} \text{ K}^{-1} \\ \mu &= 1.002 \times 10^{-3} \text{ kg/m} \cdot \text{s} \\ \rho &= 998.2 \text{ kg/m}^3 \\ c_p &= 4181.6 \text{ J/kg} \cdot \text{K} \\ K &= 0.597 \text{ W/m} \cdot \text{K} \end{aligned}$$

and from these values one calculates

$$Gr = 161.1 [R \text{ in cm}]^3 [\theta_c \text{ in } ^\circ\text{C}] \quad (3a)$$

$$Pr = 7.02 \quad (3b)$$

$$Ra = 1131 [R \text{ in cm}]^3 [\theta_c \text{ in } ^\circ\text{C}] \quad (3c)$$

Thus, with representative values of $R = 1$ cm and $\theta_c = 20^\circ \text{C}$, one has

$$Gr \approx 3220 \quad (4a)$$

$$Ra \approx 2.26 \times 10^4 \quad (4b)$$

Line Source Model

The problem of natural convection from a line source has associated with it a characteristic length L_{ch} , a characteristic time t_{ch} , and a characteristic temperature increment θ_{ch} , these being given by

$$L_{ch} = \left[\frac{\rho c_p}{g \beta Q} \right]^{1/3} \frac{\mu}{\rho} \quad (5a)$$

$$t_{ch} = \left[\frac{\rho c_p}{g \beta Q} \right]^{2/3} \frac{\mu}{\rho} \quad (5b)$$

$$\theta_{ch} = \frac{Q}{\mu c_p} \quad (5c)$$

Here Q is the heat added per unit time and per unit length of line source, μ is the viscosity, and β is the coefficient of volume expansion (increase in volume per unit volume and per unit temperature increase at constant pressure).

From the above basic characteristic quantities one can identify a characteristic velocity v_{ch} and a characteristic pressure increment p_{ch}^* , these being given by

$$v_{ch} = \left[\frac{g\beta Q}{\rho c_p} \right]^{1/3} = L_{ch}/t_{ch} \quad (6a)$$

$$p_{ch}^* = \rho \left[\frac{g\beta Q}{\rho c_p} \right]^{2/3} = \rho v_{ch}^2 = \mu/t_{ch} \quad (6b)$$

With Q taken as 3400 watts/meter (appropriate for near a single probe), and with g , β , c_p , μ , and ρ taken as enumerated following Eq. (2c), one finds

$$\begin{aligned} L_{ch} &= 0.0394 \text{ cm} \\ t_{ch} &= 0.154 \text{ s} \\ \theta_{ch} &= 8.11 \times 10^2 \text{ K} \\ v_{ch} &= 0.255 \text{ cm/s} \\ p_{ch}^* &= 0.00648 \text{ Pa} \end{aligned}$$

Alternatively, with Q taken as 8700 watts/m (appropriate for using the line source model in the description of the flow far above the heating array, one has

$$\begin{aligned} L_{ch} &= 0.0288 \text{ cm} \\ t_{ch} &= 0.083 \text{ s} \\ \theta_{ch} &= 2.08 \times 10^3 \text{ K} \\ v_{ch} &= 0.348 \text{ cm/s} \\ p_{ch}^* &= 0.0121 \text{ Pa} \end{aligned}$$

The only distinct dimensionless group that can be associated with the line source model is the Prandtl number $Pr = \mu c_p / k$. There is no intrinsic Grashof number associated with the model; the literature [7] does, however, use a coordinate dependent Grashof number. Thus, if x denotes distance above the line source,

one could speak of a Grashof number equal to $(x/L_{ch})^3$, or

$$(Gr)_x = \frac{x^3 g \beta \rho^2 Q}{c_p \mu^3} \quad (7)$$

$$= 1.63 \times 10^4 [x \text{ in cm}]^3 \quad \text{if } Q=3400 \text{ W/m}$$

$$= 4.18 \times 10^4 [x \text{ in cm}]^3 \quad \text{if } Q=8700 \text{ W/m}$$

Note that Eq. (7) corresponds to Eq. (2a) if $2R$ is replaced by x , and if θ_c is replaced by $Q/\mu c_p$.

IV. BOUNDARY LAYER NEAR A HORIZONTAL CYLINDER

Much of the recent literature [8, 9, 10, for example] on natural convection caused by horizontal cylinders has concentrated on direct numerical integration of the partial differential equations (1). Unfortunately, the results are all for air ($Pr \approx 0.7$) and are oriented toward heat transfer predictions, so an extrapolation to results applicable for the NORDA experiments is not a priori evident. However, a good approximate theory can be extracted from a boundary layer model proposed in a seldom cited sequence of papers by Merk and Prins [11]. One adopts in the vicinity of the cylinder a quasi-cartesian coordinate system (see Fig. 2) such that $x = R$ and $y = r - R$ and lets u and v be the x and y "components" of the fluid velocity. The flow is assumed steady and laminar and confined (for ϕ between 0 and, say, 0.7π) to the immediate vicinity of the cylinder. With

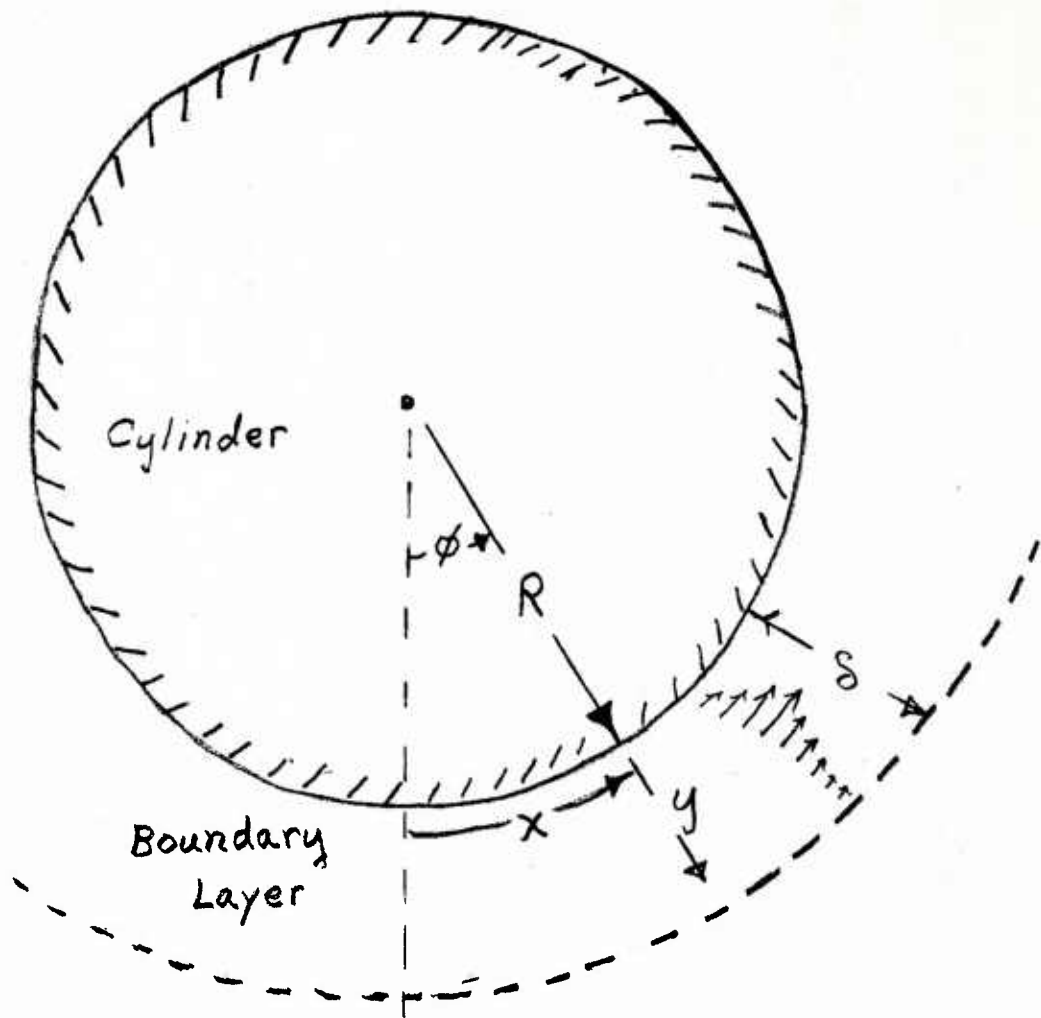


Figure 2. Boundary layer model suggested by Merk and Prins for the analysis of convective flow around a horizontal cylinder at a different temperature than the surrounding fluid.

various plausible assumptions the Boussinesq equations reduce to the boundary layer equations

$$\partial u / \partial x + \partial v / \partial y = 0 \quad (8a)$$

$$u \partial u / \partial x + v \partial u / \partial y = \nu \partial^2 u / \partial y^2 + g \theta \beta \sin \phi \quad (8b)$$

$$u \partial \theta / \partial x + v \partial \theta / \partial y = (\nu / \text{Pr}) \partial^2 \theta / \partial y^2 \quad (8c)$$

with boundary conditions $u=0$ at $x=0$; $u=0$, $v=0$, $\theta=\theta_c$ at $y=0$; and $u \rightarrow 0$, $\theta \rightarrow 0$ as $y \rightarrow \infty$. Here $\nu=\mu/\rho$ is the kinematic viscosity, which has a value of approximately $10^{-6} \text{ m}^2/\text{s}$ for water.

If u and θ are assumed to vanish outside a boundary layer of thickness δ , varying with x , then the boundary layer equations above yield the integral relations

$$(d/dx) \int_0^\delta u^2 dy = - \nu (\partial u / \partial y)_0 + g \beta \sin \phi \int_0^\delta \theta dy \quad (9a)$$

$$(d/dx) \int_0^\delta u \theta dy = - (\nu / \text{Pr}) (\partial \theta / \partial y)_0 \quad (9b)$$

Merk and Prins suggest that one assume at the outset that u and θ vary with y within the boundary layer as

$$u = F(x) (y/\delta) [1 - (y/\delta)]^2 \quad (10a)$$

$$\theta = \theta_c [1 - (y/\delta)]^2 \quad (10b)$$

where $F(x)$ and $\delta(x)$ are to be determined from Eqs. (9). Then the substitutions

$$\delta = R \left[\frac{1440}{Pr Gr} \right]^{1/4} [1 + (2/M)]^{1/4} \alpha(\phi) \quad (11a)$$

$$F = \frac{60\nu [Gr/Pr]^{1/2}}{R [1440(1 + 2/M)]^{1/2}} \Psi(\phi) \quad (11b)$$

where $\phi = x/R$, $M = (7/4)Pr$, yield the coupled ordinary differential equations

$$\alpha^2 \Psi \, d\Psi/d\phi = [M+2]\alpha^2 \sin \phi - [M+1]\Psi \quad (12a)$$

$$\Psi^2 \alpha \, d\alpha/d\phi = -[M+2]\alpha^2 \sin \phi + [M+2]\Psi \quad (12b)$$

Subsequent introduction of a function $\ddagger(\phi)$, such that

$$\alpha = 1/(d\ddagger/d\phi) \quad (13a)$$

$$\Psi = \ddagger d\ddagger/d\phi \quad (13b)$$

reduces the above system to just one differential equation

$$\frac{1}{2} \frac{d^2 \psi}{d\theta^2} = (M+2) [\sin \theta - \frac{1}{2} (\frac{d\psi}{d\theta})^2] \quad (14)$$

with boundary conditions $\psi = 0$ and $\frac{d\psi}{d\theta} = 1$ at $\theta = 0$.

The integration of the above equation, although not trivial, can be effected with the aid of a digital computer. Results are listed in Table 1 for a Prandtl number of 7.0. Insertion of the representative numbers cited earlier in the present report into Eqs. (11) yields

$$\delta/R \approx 0.52\alpha \quad (15a)$$

$$F \approx [3.15 \times 10^{-3} \text{ m/s}] \Psi \quad (15b)$$

The peak value of u in the boundary layer, according to Eq. (10a), is $(4/27)F$, so since Ψ increases to a value of the order of 1.5, the peak velocity in the vicinity of the cylinder is of the order of 7×10^{-4} m/s. The boundary layer separation point should be roughly where Ψ attains its maximum value, so one should not attach too much significance to the listings in the table for values of θ greater than 130° .

ϕ	$180\phi/\pi$	$\Phi(\phi)$	$\alpha(\phi)$	$\Psi(\phi)$
0.0	000.0	0.00000	1.00000	0.00000
0.1	5.7	0.09998	1.00048	0.09994
0.2	11.5	0.19987	1.00192	0.19949
0.3	17.2	0.29957	1.00502	0.29808
0.4	22.9	0.39898	1.00781	0.39589
0.5	28.6	0.49800	1.01217	0.49201
0.6	34.3	0.59654	1.01763	0.58620
0.7	40.1	0.69450	1.02418	0.67810
0.8	45.8	0.79178	1.03187	0.76733
0.9	51.6	0.88829	1.04075	0.85351
1.0	57.3	0.98392	1.05089	0.93627
1.1	63.0	1.07857	1.06238	1.01524
1.2	68.8	1.17214	1.07533	1.09003
1.3	74.5	1.26453	1.08984	1.16028
1.4	80.2	1.35562	1.10608	1.22561
1.5	85.9	1.44531	1.12421	1.28563
1.6	91.7	1.53348	1.14443	1.33995
1.7	97.4	1.62003	1.16701	1.38819
1.8	103.1	1.70482	1.19224	1.42993
1.9	108.9	1.78774	1.22053	1.46473
2.0	114.6	1.86864	1.25233	1.49213
2.1	120.3	1.94739	1.28827	1.51163
2.2	126.1	2.02383	1.32913	1.52267
2.3	131.8	2.09781	1.37596	1.52461
2.4	137.5	2.16912	1.43018	1.51668
2.5	143.2	2.23757	1.49376	1.49795
2.6	149.0	2.30293	1.56956	1.46724
2.7	154.7	2.36490	1.66195	1.42297
2.8	160.4	2.42314	1.77791	1.36291
2.9	166.2	2.47722	1.92967	1.28375
3.0	171.9	2.52656	2.14101	1.18008
3.1	177.6	2.57029	2.46677	1.04196

Table 1. Tabulation of dimensionless boundary layer parameters for natural convection around a horizontal cylinder at a different temperature than the surrounding fluid. Calculation is for a Prandtl number of 7.0 Here ϕ is angle in radians, $180\phi/\pi$ is angle in degrees, $\Phi(\phi)$ is the solution of the nonlinear differential equation, $\alpha(\phi)$ corresponds to boundary layer thickness, $\Psi(\phi)$ corresponds to the peak tangential fluid velocity in the boundary layer.

Analogous results can be extracted from the relations
[derived from Eqs. (10)]

$$\int_0^{\delta} u \, dy = (1/12) F \delta ; \quad \int_0^{\delta} u \, \theta \, dy = (1/30) \theta_c F \delta$$

$$\int_0^{\delta} u^2 \, dy = (1/105) F^2 \delta \quad (16)$$

concerning the rates at which mass, thermal energy, and momentum are being transported in the thermal plume.

V. LAMINAR FLOW IN THERMAL PLUME

When the fluid is locally heated, a thermal plume rises from the heated region because of natural convection; if it is locally cooled, then the thermal plume descends. In what follows, the terminology refers to a rising plume but in actuality the analysis is equally applicable to a descending plume.

The thermal plume rising from a heated horizontal cylinder or a line source may be laminar for some distance above the source but must eventually become unstable. There is possibly some intermediate range of heights (see Fig. 3) where the flow profile has a characteristic shape such that the vertical velocity u depends on horizontal distance y from the plume's center according to a relation of the generic form

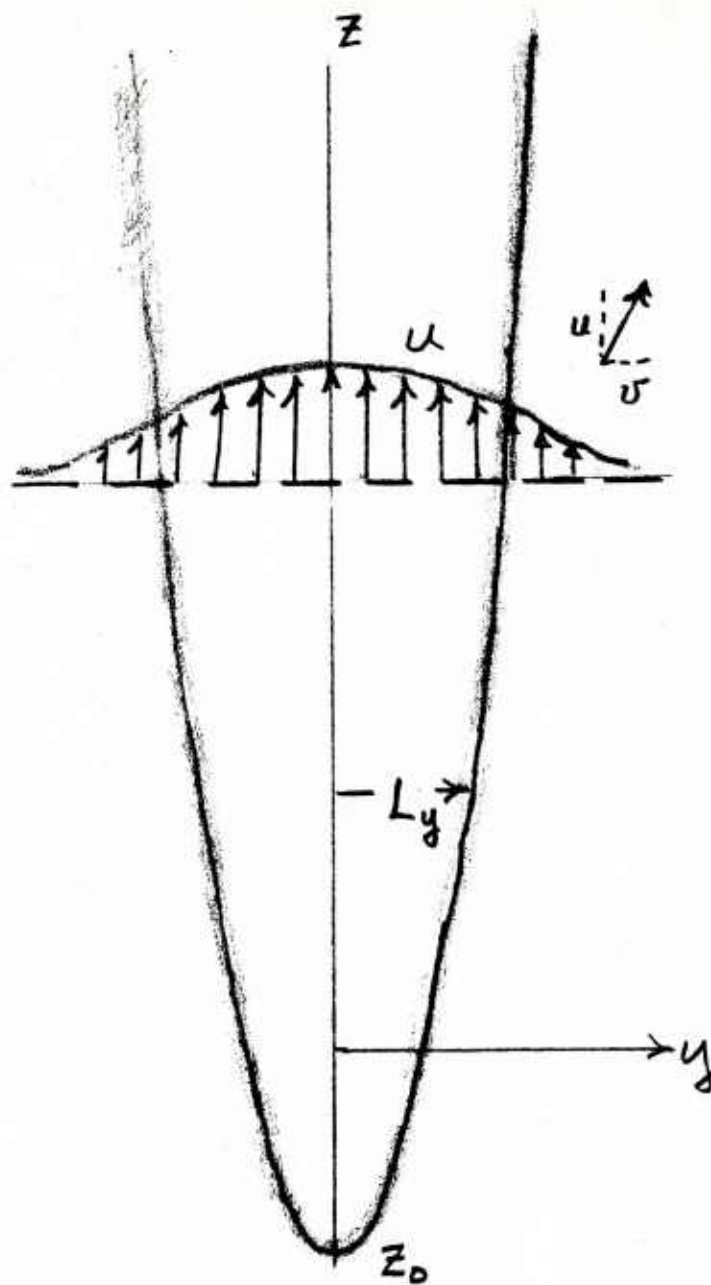


Figure 3. Sketch of thermal plume and coordinates used in the development of a similarity solution.

$$u = U(z) F(y/L_y) \quad (17)$$

where $F(\eta)$ is some characteristic function, independent of height, which can be normalized such that $F(0) = 1$. The quantities $U(z)$ and $L_y(z)$, representing peak vertical velocity in the plume and the plume's half width, will, however, vary with height z . The premise is that, given appropriate choices for $U(z)$ and $L_y(z)$, the same function $F(\eta)$ applies for each height z in this range.

Similarly, one can postulate that

$$v = V(z) G(y/L_y) \quad (18a)$$

$$\theta = T(z) H(y/L_y) \quad (18b)$$

for the transverse velocity and the temperature deviation.

Moreover, it seems plausible that the plume is still fairly thin in this height range, so boundary layer equations analogous to Eqs. (8) can be used, i.e.

$$\partial u / \partial z + \partial v / \partial y = 0 \quad (19a)$$

$$u \partial u / \partial z + v \partial u / \partial y = \nu \partial^2 u / \partial y^2 + g \beta \theta \quad (19b)$$

$$u \partial \theta / \partial z + v \partial \theta / \partial y = (\nu / Pr) \partial^2 \theta / \partial y^2 \quad (19c)$$

with the boundary conditions

$$\partial u / \partial y = 0; \quad \partial \theta / \partial y = 0; \quad v = 0; \quad \text{at } y = 0 \quad (20a)$$

$$u \rightarrow 0; \quad \theta \rightarrow 0; \quad \text{as } y \rightarrow \infty \quad (20b)$$

A corollary of the above boundary layer equations and boundary conditions is that

$$(d/dz) \int_0^{\infty} u \theta \, dy = 0 \quad (21)$$

so the functions $U(z)$, $T(z)$, and $L_y(z)$ that appear in Eqs. (17) and (18) must satisfy the requirement

$$U T L_y = K \quad (22)$$

where K is a constant independent of z .

It is readily demonstrated that the assumptions (17) and (18) are consistent with the boundary layer equations and the constraint (22) only if L_y is proportional to $(z-z_0)^{2/5}$, while U must be proportional to $(z-z_0)^{1/5}$, V must be proportional to $(z-z_0)^{-2/5}$, and T must be proportional to $(z-z_0)^{-3/5}$. Here z_0 is some constant and may be negative. Without loss of generality one can take these proportionality constants to be such that

$$U = [(g\beta K)^{2/5} / \nu^{1/5}] (z-z_0)^{1/5} \quad (23a)$$

$$V = (g\beta K)^{1/5} \nu^{2/5} (z-z_0)^{-2/5} \quad (23b)$$

$$T = K^{4/5} \nu^{-2/5} (g\beta)^{-1/5} (z-z_0)^{-3/5} \quad (23c)$$

$$L_y = \nu^{3/5} (g\beta K)^{-1/5} (z-z_0)^{2/5} \quad (23d)$$

Here the constant K must be such that

$$\int_0^\infty u \theta \, dy = K \int_0^\infty F(\eta) H(\eta) \, d\eta \quad (24)$$

the left side being presumed to be known at the outset.

The mass conservation equation is satisfied if one introduces a function ϕ and sets

$$F = d\phi/d\eta = \phi' \quad (25a)$$

$$G = (2/5)\eta\phi' - (3/5)\phi \quad (25b)$$

The remaining two boundary layer equations then yield

$$H = (1/5)(\phi')^2 - (3/5)\phi\phi'' - \phi''' \quad (26a)$$

$$-(3/5)(\phi H)' = (1/Pr)H'' \quad (26b)$$

Since $F=1$, $G=0$, and $H'=0$ at $\eta=0$, one has $\phi=0$ and $\phi'=1$ at $\eta=0$.

The second of the above two equations consequently integrates to

$$-(3/5)\phi H = (1/Pr)H' \quad (27)$$

and one consequently obtains the fourth-order nonlinear

differential equation

$$\begin{aligned} \phi'''' + (3/5)(1 + \text{Pr})\phi\phi''' + (1/5)\phi'\phi'' + (9/25)\text{Pr}\phi^2\phi'' \\ - (3/25)\text{Pr}\phi(\phi')^2 \\ = 0 \end{aligned} \quad (28)$$

with the boundary conditions

$$\phi = 0; \quad \phi' = 1; \quad \phi'' = 0; \quad \text{at } \eta = 0 \quad (29a)$$

$$\phi' \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (29b)$$

Once such a function $\phi(\eta)$ is found, the constant K can be determined from

$$\begin{aligned} \int_0^\infty u \theta \, dy &= K \int_0^\infty \phi' [(1/5)(\phi')^2 - (3/5)\phi\phi'' - \phi'''] \, d\eta \\ &= K \int_0^\infty [(\phi'')^2 + (1/2)(\phi')^3] \, d\eta \end{aligned} \quad (30)$$

The differential equation (28) can be integrated numerically, starting from $\eta = 0$, given some choice $\phi'''(0)$. If $\phi'''(0)$ is systematically varied one eventually finds one such choice that leads to a solution that satisfies the upper boundary condition. The results of such a lengthy computation are tabulated in Table 2 for $\text{Pr} = 7.0$. For small η the results are consistent with the power series expressions

η	Φ	F	H	G
0.0	0.0000	1.0000	1.0049	-0.0000
0.2	0.1989	.9842	.9400	-0.0406
0.4	0.3917	.9396	.7736	-0.0847
0.6	0.5288	.8740	.5530	-0.1075
0.8	0.6123	.7969	.3584	-0.1124
1.0	0.6879	.7161	.2220	-0.1263
1.2	0.7554	.6370	.1336	-0.1475
1.4	0.8153	.5627	.0797	-0.1741
1.6	0.8680	.4946	.0482	-0.2043
1.8	0.9143	.4333	.0300	-0.2366
2.0	0.9548	.3786	.0194	-0.2700
2.2	0.9901	.3301	.0132	-0.3036
2.4	1.0209	.2874	.0092	-0.3366
2.6	1.0477	.2499	.0067	-0.3687
2.8	1.0710	.2171	.0049	-0.3995
3.0	1.0912	.1884	.0036	-0.4287
3.2	1.1087	.1633	.0027	-0.4562
3.4	1.1239	.1414	.0020	-0.4820
3.6	1.1370	.1224	.0015	-0.5059
3.8	1.1484	.1059	.0011	-0.5281
4.0	1.1583	.0915	.0008	-0.5485
4.2	1.1668	.0791	.0006	-0.5672
4.4	1.1741	.0682	.0005	-0.5843
4.6	1.1804	.0589	.0003	-0.5999
4.8	1.1859	.0507	.0003	-0.6141
5.0	1.1906	.0437	.0002	-0.6270
5.2	1.1947	.0376	.0001	-0.6386
5.4	1.1981	.0323	.0001	-0.6491
5.6	1.2011	.0278	.0001	-0.6585
5.8	1.2037	.0238	.0001	-0.6670
6.0	1.2059	.0204	.0000	-0.6746
6.2	1.2078	.0175	.0000	-0.6814
6.4	1.2094	.0149	.0000	-0.6875

Table 2. Tabulation of similarity solution for a buoyantly rising thermal plume. Calculation is for a Prandtl number of 7.0. The dimensionless quantity η corresponds to horizontal distance across the plume relative to its center; F corresponds to the profile of vertical velocity; G corresponds to the profile of horizontal velocity (negative values imply motion away from the center); H corresponds to the temperature profile; $\Phi(\eta)$ is the solution of the differential equation and is the integral of F(η).

$$\ddagger = \eta - 0.1342 \eta^3 + 0.0335 \eta^5 \quad (31a)$$

$$F = 1 - 0.4024\eta^2 + 0.1677\eta^4 \quad (31b)$$

$$H = 1.0049 - 1.6903\eta^2 + 1.7191\eta^4 \quad (31c)$$

$$G = -0.2000\eta - 0.0805\eta^3 + 0.0470\eta^5 \quad (31d)$$

In the limit of large η the function \ddagger approaches a constant value of $\ddagger_{\infty} = 1.2173$ and the approach is such that

$$\ddagger \rightarrow \ddagger_{\infty} - (5/3)(1.698/\ddagger_{\infty}) e^{-(3/5)\ddagger_{\infty}\eta} \quad (32a)$$

$$F \rightarrow 1.698 e^{-(3/5)\ddagger_{\infty}\eta} \quad (32b)$$

$$G \rightarrow -(3/5)\ddagger_{\infty} + (3/5)[\eta + 5/(3\ddagger_{\infty})]1.698 e^{-(3/5)\ddagger_{\infty}\eta} \quad (32c)$$

The temperature profile, according to Eqs. (27) and (32a), must asymptotically satisfy

$$d(\ln H)/d\eta = -(3/5)Pr \left[\ddagger_{\infty} - (5/3)(1.698/\ddagger_{\infty}) e^{-(3/5)\ddagger_{\infty}\eta} \right]$$

so

$$H \rightarrow D e^{-(3/5)Pr\ddagger_{\infty}\eta} \exp \left[-(5/3)Pr 1.698 \ddagger_{\infty}^{-2} e^{-(3/5)\ddagger_{\infty}\eta} \right] \quad (33a)$$

and eventually

$$H \rightarrow D e^{-(3/5)Pr \frac{1}{2} \eta} \quad (33b)$$

A comparison of the expression (33a) with the numerical results yields $D = 1.07 \times 10^4$. [The asymptotic formula (33a) is probably more accurate than the Table 2 values for $H(\eta)$ when η is greater than 2 because of the round-off errors incurred when Eq. (27) is used to calculate $H(\eta)$.]

For the line source model, the energy Q generated per unit length and per unit time (see Sec. III) is identified with reference to the thermal plume boundary layer equations (19) as being

$$Q = 2 \rho c_p \int_0^\infty u \theta dy \quad (34a)$$

$$= 2 \rho c_p K \int_0^\infty H F d\eta \quad (34b)$$

where the second expression follows from Eq. (24). For a Prandtl number of 7.0 the integral appearing in this latter expression is computed to be

$$\int_0^\infty H F d\eta = 0.326 \quad (35)$$

Consequently, one has (with $Pr = 7.0$)

this matching are that (1) the rate of heat transport (integral over y of $u\theta$) and (2) the rate of momentum transport (integral over y of u^2) be the same for both solutions at the point where the boundary layer separates from the cylinder. Since the separation point is not known and since the cited integrals are somewhat insensitive to its precise value, the numbers corresponding to $\phi = \pi/2$ (90°) are used in the cylinder boundary layer solution. The z origin is taken at the center of the cylinder.

The calculations discussed here are carried out for a Prandtl number of 7.0. Consequently, the integral expressions in Eqs. (16), with substitution from Eqs. (11) and with values of $\alpha(\pi/2)$ and $\Psi(\pi/2)$ taken from Table 1, yield

$$\int_0^\infty u^2 dy = 0.0230 (v^2/R) (Gr)^{3/4} \quad (38a)$$

$$\int_0^\infty u \theta dy = 0.1095 v \theta_c (Gr)^{1/4} \quad (38b)$$

The integral on the right side of (24) is found to be 0.326, and since $0.1095/0.326$ is 0.336, one obtains

$$K = 0.336 v \theta_c (Gr)^{1/4} \quad (39)$$

Alternatively, since K is given in terms of Q by Eq. (36), this yields

$$Q = 0.219 \mu c_p \theta_c (Gr)^{1/4} \quad (40)$$

The Nusselt number for natural convective heat transfer from the horizontal cylinder can be regarded as defined by the relation

$$Nu = Q/\theta_c K \pi \quad (41)$$

Consequently, with the Prandtl number taken to be 7.0, Eq. (40) yields the heat transfer relation

$$Nu = 0.49 (Gr)^{1/4} = 0.30 (Ra)^{1/4} \quad (42)$$

which is in order of magnitude agreement with the empirical relation

$$Nu = 0.525 (Ra)^{1/4} \quad (43)$$

commonly cited in the literature [12].

The second matching criterion can be expressed

$$\begin{aligned} 0.0230 (v^2/R) (Gr)^{3/4} &= (g\beta K)^{3/5} v^{1/5} (-z_o)^{4/5} \int_0^\infty F^2 d\eta \\ &= (.336/8)^{3/5} (Gr)^{3/4} (v^2/R) (-z_o/R)^{4/5} \int_0^\infty F^2 d\eta \end{aligned}$$

so one has

$$(-z_o/R)^{4/5} = \frac{0.0230 (8/.336)^{3/5}}{\int_0^\infty F^2 d\eta} \quad (44)$$

The integral in the denominator here is 0.628, so one consequently obtains

$$z_o = -0.173 R \quad (45)$$

The apparent origin point of the thermal plume is therefore 0.173 radii below the center of the cylinder. If one is interested in the character of the plume only for z greater than, say, 5 radii, the value of z_0 can be approximated by 0.

Again, it must be pointed out that the numerical values in the above equations are for a Prandtl number of 7.0.

VII. TRANSITION TO TURBULENCE

A single buoyant plume such as is described by equations given in previous sections of this report should eventually become unstable and then become turbulent.

The earliest data in this regard of which the present author is aware appears in a 1967 paper by Forstrom and Sparrow [13]. Their experiments were with a horizontal wire heated by an electrical current; the heat input Q per unit length and per unit time in such an experiment is easily measured by simply measuring the voltage drop across a segment of length d and also measuring the current through the wire; Q is then simply voltage drop times current divided by segment length d .

Forstrom and Sparrow interpreted their data in terms of a horizontal line source model and used what they called a modified Grashof number, which turns out to be the same as the x -dependent Grashof number defined in the present report's Eq. (7). Recall that x there corresponds to height above the line source. The onset of transition to turbulence is characterized by turbulent bursts and begins when $(Gr)_x = 5 \times 10^8$. The plume appears to be fully turbulent if $(Gr)_x$ exceeds 5×10^9 .

The criteria just mentioned should also depend on the Prandtl number; Forstrom and Sparrow's experiments were carried out in air, for which $Pr \approx 0.7$, so the results are not necessarily applicable for water, for which $Pr \approx 7.0$. Nevertheless, it may be of interest to see what these criteria, taken at face value, would imply for the NORDA experiments. Using the numbers listed in Sec. III, one finds:

i) Onset of turbulence [$(Gr)_x = 5 \times 10^8$]:

$$x = 31 \text{ cm if } Q = 3400 \text{ W/m}$$

$$x = 23 \text{ cm if } Q = 8700 \text{ W/m}$$

ii) Fully turbulent [$(Gr)_x = 5 \times 10^9$]:

$$x = 67 \text{ cm if } Q = 3400 \text{ W/m}$$

$$x = 49 \text{ cm if } Q = 8700 \text{ W/m}$$

A second paper which pertains to this question is that published in 1971 by Pera and Gebhart [14]. Their discussion uses a different definition [15] of a Grashof number, which we abbreviate here as $(Gr)_{PG}$, the definition being such that

$$(Gr)_{PG} = \frac{g \beta x^3 (\Delta\theta)_x}{(\mu/\rho)^2} \quad (46)$$

with

$$\begin{aligned} (\Delta\theta)_x &= \theta(x,0) - \theta(x,\infty) \\ &= \theta(x,0) \end{aligned} \quad (47)$$

representing the plume's nominal center temperature (relative to the ambient temperature) at height x above the line source.

Given that the plume is on the borderline of being instable one can take the latter quantity to be as given by Eq. (18b), only

with x now denoting the vertical distance rather than $z-z_0$. The function H is evaluated with its argument set to zero; the corresponding value for when $Pr=7.0$ is $H(0) = 1.0049$ from Eq. (31c). Consequently, with K given by (36), and with $T(x)$ given by Eq. (23c), one has a reinterpretation of the Pera and Gebhart Grashof number as being (for a Prandtl number of 7.0)

$$\begin{aligned} (Gr)_{PG} &= (1.53)^{4/5} H(0) [(Gr)_x]^{4/5} \\ &= 1.41 [(Gr)_x]^{4/5} \end{aligned} \quad (48)$$

Most of Pera and Gebhart's computations and experimental results are for air, for which $Pr = 0.7$, but their Figs. 2, 3, and 4 indicate no radical dependence on Prandtl number in regard to stability criteria. A rough inference is that if $Gr_{PG}(\text{air})$ is some critical value of a Grashof number for air, then the corresponding critical value $Gr_{PG}(\text{water})$ is between 1 and 10 times $Gr_{PG}(\text{air})$, larger but no more than 10 times larger. They report a "critical" Grashof number Gr_{PG} of 10.34 for air, so it is a good conclusion that any plume in water must be intrinsically unstable whenever Gr_{PG} exceeds 103.4, or with reference to Eq. (48), whenever $(Gr)_x$ exceeds 215. This is considerably smaller than Forstrom and Sparrow's value of 5×10^8 , but one could possibly explain this with the supposition that a very small instability triggered at a critical value of x may not be noticeable until the plume has risen some additional distance. Recall that $(Gr)_x$ is proportional to x^3 , so a 10^6 -fold increase in $(Gr)_x$ only corresponds to a 100-fold increase in x . For the planning of experiments, Forstrom and Sparrow's number should be the more useful.

VIII. TURBULENT PLUMES

A plume rising from a line source must eventually become fully turbulent. In such a height regime the mean flow and mean temperature are more strongly affected by Reynolds' stresses than viscous stresses and more strongly affected by heat transport by turbulent diffusion than direct thermal conduction. Consequently, the plume boundary layer equations (19) must be modified to [16]

$$\partial u / \partial z + \partial v / \partial y = 0 \quad (49a)$$

$$u \partial u / \partial z + v \partial u / \partial y = (1/\rho) \partial \tau / \partial y + g \beta \theta \quad (49b)$$

$$u \partial \theta / \partial z + v \partial \theta / \partial y = (1/\rho c_p) \partial \chi / \partial y \quad (49c)$$

where here the quantities u , v , θ , correspond to time averaged quantities. Apart from a multiplicative factor of ρ , the Reynolds' stress τ is presumed to depend only on height z and on the local profile of u versus y at that height. Similarly, apart from a multiplicative factor of ρc_p , the mean thermal heat flux χ (which is associated with turbulent convection) is presumed to depend only on height z and on the local profiles of u and θ with the transverse coordinate y . It is still so that the quantity

$$Q = \rho c_p \int_{-\infty}^{\infty} u \theta \, dy \quad (50)$$

must be independent of z and is identifiable as the net heat per unit source length and per unit time that is being carried upward by the plume.

A plausible assumption that can be made concerning the solution of Eqs. (49) is that at sufficiently large heights the mean plume profile must eventually attain a self similar form, such as is described by Eqs. (18), with L_y being some function of z . A related assumption is that the shear stress τ must scale with ρU^2 , such that $\tau/\rho U^2$ is a function only of y/L_y . Existence of a solution having such properties requires [17] that U be independent of z , that θL_y be a function only of y/L_y , and dL_y/dz be independent of z . Hence with a suitable choice of the z -origin, one can set $L_y = z/a$, where a is any convenient dimensionless constant. The conservation of mass equation (49a) is automatically satisfied if one introduces a function $\phi(\eta)$, where

$$\eta = ay/z \quad (51a)$$

$$u(y,z) = [g\beta Q/\rho c_p]^{1/3} K\phi' = v_{ch} K\phi' \quad (51b)$$

$$v(y,z) = [g\beta Q/\rho c_p]^{1/3} a^{-1} K[\phi'\eta - \phi] \quad (51c)$$

Here K and a are dimensionless constants that can be selected to make the analysis less cumbersome; the quantity v_{ch} is as defined in Eq. (6a).

When the similitude principle is applied to the other two boundary layer equations one discovers that

$$\tau = \rho [g\beta Q / \rho c_p]^{2/3} (K^2/a) A_\tau \quad (52a)$$

$$\theta = [g\beta Q / \rho c_p]^{2/3} (K^2/g\beta z) A_\theta = (\theta_{ch} L_{ch}/z) K^2 A_\theta \quad (52b)$$

$$x = QK^3 a^{-1} A_x \quad (52c)$$

where A_τ , A_θ , and A_x are dimensionless functions of η ; the quantities L_{ch} and θ_{ch} are as defined in Eqs. (5a) and (5c).

Insertion of the above expressions into the boundary layer equations (49b) and (49c) consequently leads to the two ordinary differential equations

$$-\frac{1}{2} \frac{d^2 \psi}{d\eta^2} = A'_\tau + A_\theta \quad (53a)$$

$$-\frac{1}{2} [A_\theta]' = A'_x \quad (53b)$$

the latter of which immediately integrates to

$$-\frac{1}{2} A_\theta = A_x \quad (53b')$$

because symmetry imposes the boundary conditions $\psi(0) = 0$ and $A_x(0) = 0$. A comparable insertion into Eq. (50) yields the relation

$$1 = (K^3/a) \int_{-\infty}^{\infty} \frac{1}{2} A_\theta d\eta \quad (54)$$

which can be used for the evaluation of the constant K , given some choice for the value of $\psi'(0)$.

The Prandtl model of turbulence requires that

$$A_r = c^2 a^3 \phi''' | \phi''' | \quad (55a)$$

$$A_x = c^2 a^3 A'_\theta | \phi''' | \quad (55b)$$

where c is a dimensionless constant that must be determined from experiment. [A determination described further below yields $c = 0.065$.] For a hot rising plume, given positive y , ϕ''' and A'_θ should be negative.

The most convenient choice for the parameter a is such that $c^2 a^3 = 1/2$. Such a choice along with the identifications in Eqs. (55) allows one to reexpress Eqs. (53) in the form (for positive η)

$$\phi''' \phi'''' - \phi \phi''' = A_\theta \quad (56a)$$

$$\phi''' A'_\theta - 2\phi A_\theta = 0 \quad (56b)$$

Boundary conditions at $\eta=0$ are that both ϕ and ϕ''' are zero. The quantity ϕ' should be finite at $\eta=0$. Given the latitude in the selection of K , one is free to choose the solution to be such that $A_\theta(0) = 1$. Such conditions cause the solution of Eqs. (56) to be such that, for small positive η ,

$$\phi = C\eta - (4/15)\sqrt{2}\eta^{5/2} + (1/18)C\eta^4 - \dots \quad (57a)$$

$$A_\theta = 1 - (2/2/3)C\eta^{3/2} + [(2/9)C^2 + (8/45)]\eta^3 - \dots \quad (57b)$$

where C is a constant that remains to be determined.

The quantities u and θ must vanish at large distances from the center of the plume; a study of Eqs. (56) indicates that this can be achieved if ϕ' and ϕ'' simultaneously vanish at some point. If such a point is denoted by η_L , then the differential equations and the requisite continuity conditions are identically satisfied beyond η_L with ϕ set to a constant value of $\phi(\eta_L)$. There is, however, only one such value of the constant C for which this is possible; Schmidt's calculations [16] yield

$$C = 1.1398 \quad (58a)$$

$$\eta_L \approx 2.0 \quad (58b)$$

$$\phi(\eta_L) = 0.9682 \quad (58c)$$

The plume therefore stops abruptly at $\eta = \eta_L$ and has a finite width. A brief table of values extracted from Schmidt's paper is given in Table 3.

The experiments of Rouse, Yih, and Humphreys [17] yield

$$u(0,z) = 1.80 (g\beta Q/\rho c_p)^{1/3} \quad (59)$$

so a comparison with Eqs. (51b), (54), and (57a) requires

$$K = 1.80/C = 1.80/1.1398 = 1.58 \quad (60a)$$

$$a = 2(1.80/1.1398)^3 \int_0^{\eta_L} \phi' A_\theta d\eta = 4.93 \quad (60b)$$

$$c = (1/2a^3)^{1/2} = 0.065 \quad (60c)$$

η	ψ	ψ'	A_θ
0.0	0.00000	1.1398-	1.00000
0.1	0.1128	1.1202	0.9665
0.2	0.2215	1.0582	0.90666
0.3	0.3154	0.9888	0.8311
0.4	0.41995	0.91698	0.75703
0.5	0.507	0.8369	0.67354
0.6	0.5866	0.7607	0.59068
0.7	0.65775	0.66914	0.51964
0.8	0.7204	0.58575	0.4324
0.9	0.7749	0.5032	0.387
1.0	0.8205	0.4246	0.294
1.1	0.8604	0.3525	0.2365
1.2	0.8915	0.2812	0.1818
1.3	0.9166	0.2185	0.1364
1.4	0.9356	0.1622	0.0983
1.5	0.9495	0.1139	0.0678
1.6	0.9584	0.0733	0.0496
1.7	0.9639	0.0415	0.0233
1.8	0.9667	0.0178	0.009
1.9	0.9678	0.0048	0.0025
2.0	0.9682	0.0000	0.0000

Table 3. Tabulation of dimensionless boundary layer parameters for a turbulent plume, buoyantly rising under the influence of gravity. Calculation applies to two-dimensional plume, such as would be generated by a line source of heat, and is for a Prandtl number of 7.0. The parameter η is ay/z and is proportional to transverse distance y ; ψ is the solution of an ordinary differential equation, defined such that ψ' corresponds to vertical flow velocity; A_θ corresponds to temperature.

where the numerical integration in Eq. (60b) is effected using the values from Table 3.

Since the temperature deviation drops to 1/2 its maximum value at $\eta \approx 0.7$, the plume width can be taken to be $(2)(0.7)a^{-1}z$, or with the numbers derived above, 0.28 times the height z above the heat element. Thus at a representative height of 2 m, the plume width should be roughly 0.57 m. The heuristic mixing length theory analyses that lead to Eqs. (55) suggest that the correlation length ℓ that appears in Sec. I of the present report should be roughly cz , or 0.06 times the height z above the heating element ($\ell = 12$ cm when $z = 2$ m).

Given that Q is 8700 m/s, along with the numbers cited in Sec. III, one finds that the above analysis leads to

$$\theta_{\max} = 1.5/z \quad (61)$$

for the temperature in the center of the plume (in degrees centigrade relative to the nominal temperature of the water when z is measured in metres). Since the speed of sound in water increases by roughly 4.0 m/s when the temperature increases by 1 °C, the ratio v^2 of the mean squared deviation of the sound speed from its average value to the square of the average sound speed should be roughly of the order of

$$\begin{aligned} v^2 &\approx (4.0/1500)^2 \theta_{\max}^2 \\ &\approx (1.6 \times 10^{-5})/z^2 \end{aligned} \quad (62)$$

with all numerical values understood to be reckoned in MKS units. Given that $\ell \approx 0.06z$, one consequently has

$$v \approx (10^{-6})/z \quad (63)$$

IX. PLUME-PLUME INTERACTIONS

One of the features that shows up in interferometer photographs [18] of multiple plumes from parallel line sources lying in the same horizontal plane is that the separate plumes tend to draw themselves together. The tentative explanation of this phenomena is that the upward moving fluid in any given plume drifts sideways because of the asymptotic horizontal velocity of the other plumes. The bending can be worked out starting with this premise and tracing the trajectory of a fluid particle nominally in the center of the plume, according to the equation

$$dy/dz = \frac{v \text{ due to other plumes}}{u \text{ due to that plume}} \quad (64)$$

Using the equations developed in Section V of the present report, one has in the case of two plumes

$$dy/dz = \frac{v^{3/5} (3/5) (1.2173)}{(g\beta K)^{1/5} (z-z_0)^{3/5}} \quad (65)$$

For the horizontal isothermal cylinder model this gives [see Eqs. (39) and (45)]

$$dy/dz = \frac{1.38}{(Gr)^{1/4}} [z/R + 0.173]^{-3/5} \quad (66)$$

Thus, in the limit of large z , one has

$$y \approx \frac{3.44 (z/R)^{2/5}}{(Gr)^{1/4}} R \quad (67)$$

for the sideways drift y of one of the plumes. With $z/R = 10$

and $Gr = 3000$, for example, one would have $y/R = 1.2$, while for $z/R = 100$ one would have $y/R = 3$, and for $z/R = 1000$ one would have $y/R = 7$.

X. SWAYING OF PLUMES

A curious feature of the thermistor data from experiments in the NORDA water tank facility is that there are often relatively long "quiet" periods in which the temperature does not fluctuate at all at the measurement point. Although there is no clear-cut unambiguous explanation for this phenomenon, a possibility suggested by a study of the recent archival literature on natural convection is the "swaying of thermal plumes."

The "swaying" could cause the "quiet" periods if the turbulent plume arising from the heating element were confined to a relatively narrow region of horizontal extent. Part of the time the sensor may lie in the plume; part of the time the plume may have swayed off to one side of the sensor; part of the time it may have swayed off to the other side.

Present understanding of the physics of plume swaying is unfortunately relatively meagre, but something may perhaps be learned from a careful study of previous literature on this subject. With such a purpose, the relevant portions of the principal papers are paraphrased and discussed below.

A. Forstrom and Sparrow (1967)

This early paper [13] summarizes experiments on the buoyant plume above a heated horizontal wire. The experiments were done in air with a wire 0.040 inch in diameter. In discussing

the design aspects of their experiments, the authors state that "prior experience suggests that external free convection flows are readily affected by fluid motions and temperature fluctuations in the surroundings", so they were careful to use an isolated, windowless room, free of drafts. Nevertheless, they found that in any such room, the measured temperatures varied appreciably with time. Subsequent visualization of the flow field with a Schlieren system revealed that the plume was swaying to and fro in a plane perpendicular to the axis of the wire. The authors concluded from this that even in the ideal rooms available, there were sufficient air currents to affect the plume. To eliminate such spurious air currents, they surrounded the plume with an isolation enclosure. This enclosure was a rectangular box 44-in high with 32 x 24-in horizontal dimensions, the 32-in length being parallel to the heated wire. The wire itself was centrally located 12-in above the bottom of the enclosure. Individual experiment runs were distinguished by the amount of heat Q dissipated per unit time and per unit length of wire. This was measured by determination of the time average of the product of voltage drop across the wire times current through the wire, divided by length of the wire. The temperature above the heated wire was measured by a thermocouple. The heating rate Q ranged from 0.26 to 5.2 W/ft. The temperature field was measured at elevations ranging from 1/4-in to 15-in above the center of the heated wire. The authors' enclosure appreciably reduced the swaying of the plume, but some swaying nevertheless persisted. These swaying motions were regarded as a characteristic of natural convection plumes. The

flow was always fully laminar for all operating conditions at elevations up to 6 in above the wire. At higher elevations, depending on heating rate, the flow would sometimes be turbulent. The swaying of the plume was manifested by a periodic oscillation in the temperature at a fixed point above the wire where the flow was laminar. The period was of the order of 1 minute. Turbulence was distinguished from laminar flow by the fact that the temperature fluctuations were erratic rather than smoothly periodic.

The authors do not give much information regarding the turbulent bursts that they report as characterizing height regimes where the flow starts to become turbulent. They state that measurements of temperature fluctuations versus time during such circumstances could not be performed because the available instrumentation did not possess the requisite rapid response characteristics, but the amplitudes of the fluctuations were much higher than those of the temperature fluctuations in the laminar regime. There is no mention of long quiet periods between turbulent bursts; instead there are remarks to the effect that the plume becomes wider when the flow becomes turbulent.

B. Aiba and Seki (1976)

This [19] was the first systematic study of the swaying motion of plumes. The heating elements were strips of stainless steel foil, each sample being 40 cm long, the width being either 1, 2, or 3 cm. Like Forstrom and Sparrow, these authors also measured the rate Q of heat supplied per unit length of strip. Temperature was measured at a point 3 to 4 cm vertically above the center of the strip. The horizontal heating strip was immersed

in transformer oil or spindle oil (the authors vaguely state that the Prandtl number varied from 80 to 160 for their experiments).

The authors' primary discovery was the manner in which the plume swaying frequency varied with the height H of the free surface of the oil above the heating strip. Their data is fairly well explained by the empirical formula

$$f d^2/\nu = 1.37 \times 10^{-3} (Gr d/H) \quad (68)$$

where f is the swaying frequency of the plume, d is the width of the heating strip, ν is the kinematic viscosity, H is distance of free surface above the heating strip, and Gr is a Grashof number. The latter is not explicitly defined in the paper but is here evidently

$$(Gr)_d = (d/\nu)^3 g \beta Q / \rho c_p \quad (69)$$

which is just the same as that defined in Eq. (7), only with x replaced by d . If one plugs (69) into (68) one finds that the swaying frequency f is actually independent of the heating foil's width d , which makes sense because H is so much larger than d , so the swaying frequency is about the same as if the source were a wire of indeterminate small diameter. What is more interesting about the empirical results is that the swaying frequency varies as the inverse square root of the fluid depth H . The deeper the fluid, the longer the period of the plume swaying. It may be somewhat risky, however, to assume that these results apply to the NORDA experiments, because the Prandtl number of

water is only 5 to 10% of that of the fluids used by Aiba and Seki. The authors suggest (but give nothing in the way of theory) that the plume swaying might be a "self-excited oscillation" related to a periodical variation of local heat transfer on the surface.

C. Eichhorn and Vedhanayagam (1982)

This [20] is a combined experimental and theoretical study of plume swaying in a rectangular tank partially filled with water. As in the experiments of Forstrom and Sparrow, these experiments used a long cylindrical wire carrying electric current to serve as a line source of heat (Q watts per unit length of wire); the new feature of the experiments was that the lateral extent of the tank (distance between plexiglas endplates) was varied in addition to Q and the depth h of the wire below the water surface. This lateral extent L corresponded to the line source length.

The flow visualization techniques in this paper were relatively novel. The outer steel sheath of the heater element acted as a dye-producing element for thymol-blue indicator which made the thermal plume visible to the eye and to the camera. Goodyear Pliolite VT particles were first used to visualize the flow field; but the latter experiments used ground-up mother of pearl as it allowed the authors to visualize both the plume and the ambient motion and to determine the swaying frequency with a stop watch. They could illuminate either horizontal or vertical thin cross sections of the flow field with collimated light from a slide projector.

The photographs of horizontal cross sections of the plume (such as would be seen looking down from above with only the por-

tion of the plume at a given set height visible) showed a wavy appearance reminiscent of the vibration modes of a vibrating string with free ends. (My inspection of the photographs in the paper doesn't lead to any simple boundary condition at the end plates. Neither the plume displacement nor the plume slope seem to vanish. What may possibly be the case is that the average plume displacement from the line directly above the source averages out to be zero.)

When the plume is swaying back and forth at constant frequency the plume shape tends to remain fixed - just as the shape of a vibrating string remains fixed when the string is vibrating in a given natural mode. Plume shapes are distinguished by the number of nodes. The authors state that "in steady state oscillation, an integral number of nodes must exist between the end plates, and the end plates must coincide with an anti-node, to satisfy continuity in the ambient fluid." Their plume visualization studies indicate that the number of such nodes is a function only of the parameter gh^3/v^2 , where h is the immersion depth of the heat source. Interestingly, the number of nodes is independent of the rate Q of heat added per unit length of line source. The more shallow the immersion, the greater the number of nodes. The change from 2 to 3 nodes occurs when the parameter gh^3/v^2 decreases from above to below 1650; the change from 3 to 4 nodes occurs when this parameter decreases from above to below 1350; the change from 9 to 10 nodes occurs when the parameter is of the order of 660 to 690.

For the NORDA experiments, the length h is of the order of

3.5 m, while ν is of the order of $1.004 \times 10^{-6} \text{ m}^2/\text{s}$, so

$$g h^3 / \nu^2 \approx 4.2 \times 10^{14} \quad (70)$$

The exceptionally large value of this number compared to the values cited in the preceding paragraph would seem to imply that there should only be 2 nodes in any swaying plume that may exist in the NORDA experiments. (Apparently, two is the minimum number of nodes that one can have, in order that "the end plates coincide with anti-nodes.")

With a given observed plume shape function one can associate a wavelength λ . This wavelength is twice the distance between nodes; the line source length L should be an integral number of half wavelengths. With such a definition, the authors present a relatively crude theory that states among other things that the swaying frequency depends, among other things, on the wavelength λ . In particular, they predict, by using a novel and somewhat unorthodox interpretation of Rayleigh's principle, that the swaying frequency is given by

$$f = (g^2/\nu)^{1/3} (\beta Q/K)^{0.3} Pr^{-0.36} (gh^3/\nu^2)^{-11/30} \text{ times (function of } h/\lambda) \quad (71)$$

where they offer two alternate expressions for the function of h/λ that appears here. These are

$$\text{function of } h/\lambda = 0.7247 \left[\frac{h/\lambda}{1 + (\lambda/2\pi h)^2} \right]^{1/2} \quad (72a)$$

and

$$\text{function of } h/\lambda = 0.4678 \left[\frac{h/\lambda}{1 + (3/2)(\lambda/2\pi h)^2} \right]^{1/2} \quad (72b)$$

In the above formulas the various symbols have their usual meanings: k is the thermal conductivity, Pr is the Prandtl number (viscosity times specific heat divided by thermal conductivity), h is depth of line source below free surface, ν is kinematic viscosity, β is coefficient of thermal expansion.

The agreement of theory and experiment is pretty good when the first version to the function of h/λ is used in Eq. (71). Regardless of whether or not the theory is correct, however, it does not begin to purport to explain the variation of the number of nodes with the parameter gh^3/ν^2 .

Applying the above formulas to the NORDA experiments is intrinsically questionable, but a rough interpretation might take $\lambda = 10$ m, $h = 3.5$ m, $Q = 8700$ W/m. The function given by Eq. (72a) is then 0.39; this in turn when inserted into Eq. (71) yields a frequency of 1.35×10^{-4} Hz, which corresponds to a period of 2 hours.

D. Urakawa, Morioka, and Kiyota (1983)

This somewhat parochial paper [21] refers only to previous Japanese work. The authors apparently were unaware of the work discussed above by Eichhorn and Vedhanayagam, but the general experimental setup is similar to that of those authors. Urakawa and coworkers used spindle oil rather than water because spindle oil has a much higher viscosity than water, and this feature evidently causes the swaying period to be much longer. There was

a horizontal line source a distance H below the free surface; the length L of the line source was varied with movable end plates (partition plates within a larger tank).

The heat rate Q per unit length of line source was measured and the time variation of the temperature with a thermocouple at some unspecified point above the heat source. The plume vibrations were visualized by shining light up from below the bottom of the tank. Between the fluorescent light source and the transparent bottom of the tank was a milk white plate with many parallel black lines painted on it. The black line below the plume would have an apparent displaced position because of the refraction of the light in passing through the heated fluid with a smaller density and different index of refraction. The swaying motion of the plume was therefore displayed in the swaying of the image of the black line.

One general observation of Urakawa et al. that conflicts with the results reported by Eichhorn and Vedhanayagam is that the position of the nodes move and the ripples in the swaying plume meander. The plume motion is not generally periodic. However, it was evidently always possible to associate a definite wavelength with the ripple in the plume; the nodes might meander but the distance between them stayed fixed. This wavelength depended on Q and the immersion depth h , but was apparently independent of the line source length L (distance between the movable partitions). They found that they could stop the meandering if they adjusted L such that it was an integral number of half wavelengths.

Another curious feature suggested by these experiments is that the line formed by the intersection of the plume with a vertical plane perpendicular to the line source has an intricate motion. Such lines were visualized by putting aluminum powder in the oil. Short term exposures of photos taken end on of the tank revealed the streak lines of the paths of the powder particles. The longer streaklines evidently marked where the flow velocity was highest and this was apparently identified as the position of the plume. A succession of such photographs then gave the time evolution of a given vertical slice of the plume. The authors' sketches derived in this manner show a line moving in a very erratic manner, sometimes bending downwards, then bending back upwards, sometimes actually becoming disconnected at an intermediate point between the line source and the free surface. On the two sides of this erratically moving plume were many (say, four to six) vortices distinguished by closed streak lines, these vortices having no discernible regularity.

The authors' measurements apparently indicated that there was more than one possibility for the wavelength λ , given a fixed choice for the immersion depth h and the heating rate Q . The swaying period can be regarded as a function of λ , h , and Q , with the wavelength in turn being one of a relatively small number of functions of h and Q . For fixed Q and h , the period increases with the wavelength. For fixed Q , the wavelength increases with the water depth and, for fixed depth and wavelength, the period varies as the reciprocal of the cube root of the heat rate Q .

The authors do not give anything in the way of theory to explain

their experimental results. Also, an extrapolation to water is hampered because the authors do not give their results in terms of dimensionless groups of parameters. The paper is stimulating but not rigorously written.

One possible explanation of the swaying of plumes in the NORDA water tank is that the the plume is being moved bodily by one of the sloshing modes (gravity induced oscillations marked by wave motion on the surface) of the tank as a whole. It is a straightforward exercise in fluid mechanics to derive an expression for the natural sloshing frequencies of a rectangular tank with a free surface. The general result is somewhat cumbersome, so it is omitted here. The pertinent prediction is that the lowest natural sloshing frequency is of the order of $(2\pi)^{-1}(g/H)^{1/2}$, where H is a characteristic dimension for the tank. Typically, this would correspond to periods of less than 20 s, although longer intervals of quiet are observed in the NORDA facility.

XI. CONCLUDING REMARKS

Turbulent natural convection, like all other aspects of turbulence, is a subject that is imperfectly understood. Nevertheless, it is still possible to make qualitative and even quantitative, although imprecise, predictions relative to experiments in the NORDA high-frequency scattering facility. Given the many simplifying assumptions in the analysis presented here, it is encouraging that the predicted rms temperature fluctuations in Sect. VIII are in rough order of magnitude agreement with Posey's oral statement [3] that the measured values were of the order of 0.1°C . Consequently,

it is a reasonable conjecture that the analysis in that Section may be a useful guide as to the dependence of the turbulent field on the externally controllable experimental parameters. In particular, one may note the dependence on Q (to the $2/3$ -rds power) in Eq. (52b). The present analysis suggests that the experimental configuration most recently reported [3] is just on the borderline of displaying Wenzel's [5] saturation phenomenon. To increase the strength of the turbulence one can either increase the number of heating elements or the voltage drop across the heating array.

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APPENDIX A

A Chronological Bibliography of Literature on Natural Convection That may Pertain to the NORDA High-Frequency Bound Scattering Facility

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